Disturbance Rejection Analysis of a Disturbance Observer Based Velocity Controller

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***Abstract*- Motion control usually includes the precise control of position/velocity and acceleration control while accurate acceleration control is not useful in many applications. Traditional position/velocity control is derived by feedback based, PID controllers. However the disturbance observer further increases the stability and robustness of the controller. In this paper we assess the effectiveness of the Disturbance Observer working together with a tuned PID controller. Velocity response is compared with and without the disturbance observer. The proposed disturbance observer based velocity controller produce better results than the traditional velocity controller.**

Keywords-component; disturbance observer;velocity controller; dc motor controlling.

I. INTRODUCTION

Although there are many numbers of motors like AC motors, stepper motors etc., DC motors are widely used due to their easiness of controlling, simplicity and availability [1]. DC motor control can be accomplished by means of minor number of components. One of the very popular methods for controlling a DC motor is velocity controlling with feedback and PID controller [2]. Since most of digital controllers have PWM outputs, voltage controlling is achieved by means of the PWM. Fig. 1 shows a simple block of a PID based velocity controller of a DC motor.

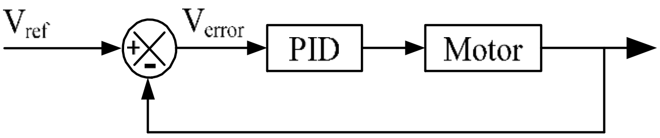

Figure. 1 Basic PID based Velocity controller for DC motor

In order to analyze the behavior of the velocity control system depicted in Fig. 1 under various conditions the system is needed modeled mathematically .

A. DC Motor Model

Fig. 2 shows the Electrical Model of a DC motor where R_a and L_a represents the armature resistance and inductance respectively.

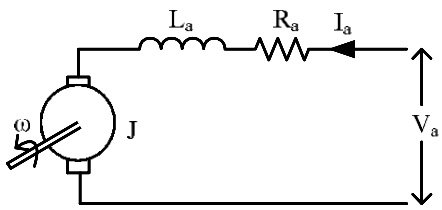

Figure. 2 Electrical Model of a DC motor

Applying Kirchhoff's voltage law the the DC motor model (1) can be obtained.

$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + E_b \quad (1)$$

Where E_b is the back emf.

$$E_b = K_e \omega(s) \quad (2)$$

Where K_e is the back emf constant, then

$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + K_e \omega(s) \quad (3)$$

If the total mechanical output given by the motor is T_m

$$I_a = \frac{T_m}{K_t} \quad (4)$$

Where k_t is the torque constant of the motor.

Applying (4) in (3) gives (5),

$$V_a = L_a \frac{d(T_m / K_t)}{dt} + R_a \frac{T_m}{K_t} + K_e \omega(s) \quad (5)$$

Motor torque T_m can be written as follows,

$$T_m = J \frac{d\omega}{dt} + B\omega + T_l + T_f \quad (6)$$

Where,

J- Inertia of the Motor
B-Viscous friction Coefficient
T_l – Load Torque
T_f- Static Friction

$$V_a = Js\,\omega(s) L_a s + Js$$

$$\omega(s) = \frac{T(s) - \left[T_l(s) + T_f(s)\right]}{Js + B} \tag{7}$$

$$T(s) = K_t I_a(s) \tag{8}$$

Where K_t is the torque constant.

II. MODELING AND ANALYSIS

A. PID based Velocity Controller

Velocity can be controlled using the voltage as shown in Fig. 3 [3]. Further according to (6) the motor torque can be controlled by controlling the current. Fig. 4 shows a block diagram of a PID based velocity controller where electrical and mechanical relationships can be clearly noticed. For velocity control, back emf also plays an important role as feedback. The same PID based velocity controller was used for integrating the Disturbance Observer.

$$V_a = I_a\left(Ls + R_a + \frac{K_e K_t}{Js + B}\right) - \frac{K_e(T_l + T_f)}{Js + B} \tag{9}$$

This PID based velocity controller works well when there are no external disturbances and the results are presented in the results section. Moreover results have also been presented after introducing Disturbance Observer [4] to the system. Then these two results are compared to validate the applicability of the proposed method.

Fig. 4 can be simplified to the form shown in Fig. 5 using the principal of superposition. S(s) is the output's sensitivity to the disturbance. G(s) is the system's transfer function. In other words S(s) and G(s) is the contribution by disturbance and the input respectively to the output.

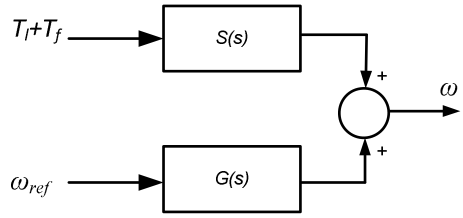

Figure 5. System Equivalent with Superposition

For the analysis viscous friction coefficient has been neglected since the external disturbance and the friction is much higher than the viscous friction in most of practical cases. Applying superposition theorem to block diagram in fig. 5 and when T_{dis} = 0, (10) can be obtained.

$$\omega = \omega_{ref}\, G(s) \tag{10}$$

and when ω_{ref} = 0, (11) can be obtained.

$$\omega = T_{dis}\, S(s) \tag{11}$$

Applying superposition theorem to block diagram in Fig. 4 and when T_{dis} = 0 (12) can be obtained.

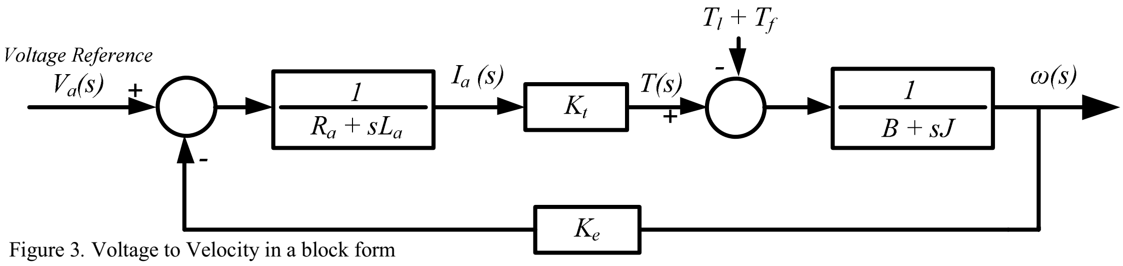

Figure 3. Voltage to Velocity in a block form

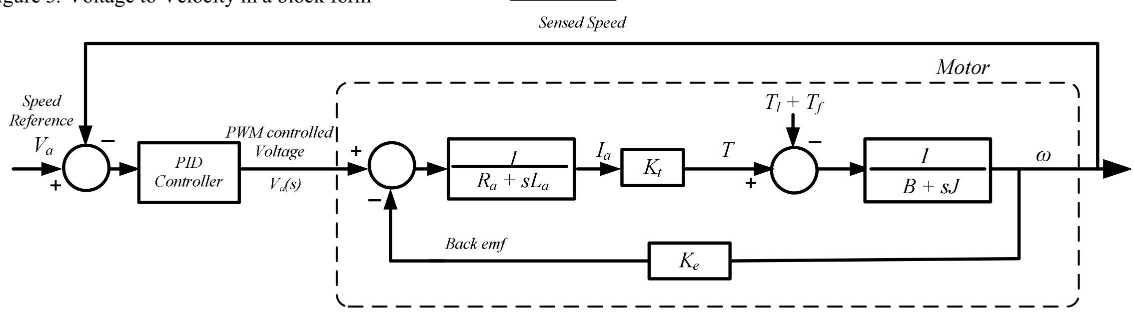

Figure 4. Velocity controller with PID

$$\omega(s) = \omega_{ref}\left[\frac{K_t\left(S^2K_d + SK_p + K_i\right)}{S^3JL_a + S^2(JR_a + K_dK_t) + S(K_eK_t + K_pK_t) + K_iK_t}\right] \quad (12)$$

Applying superposition theorem to block diagram in Fig. 4 and when $\omega_{ref} = 0$, (13) can be obtained.

$$\omega(s) = T_{dis}\left[\frac{S^2L_a + SR_a}{S^2(L_a - K_tK_a) + S(R_a - K_tK_p - K_tK_e) - K_tK_i}\right] \quad (13)$$

Comparing 10 to 12 and 11 to 13, G(s) and S(s) can be found

$$G(s) = \frac{K_t\left(S^2K_d + SK_p + K_i\right)}{S^3JL_a + S^2(JR_a + K_dK_t) + S(K_eK_t + K_pK_t) + K_iK_t} \quad (14)$$

$$S(s) = T_{dis}\left[\frac{S^2L_a + SR_a}{S^2(L_a - K_tK_a) + S(R_a - K_tK_p - K_tK_e) - K_tK_i}\right] \quad (15)$$

Disturbance's influence to the output angular speed is described by (14) while reference input's influence is described by (13). When analyzing sensitivity function in frequency domain, it can be noticed that sensitivity to each and every frequency component of the disturbance can't be eliminated by adjusting PID parameters.

B. Disturbance Observer

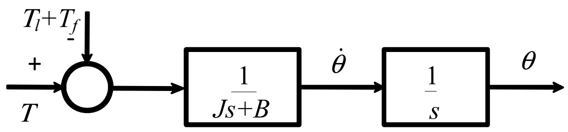

Figure 6. Block diagram of a servo motor

When a servo motor with one degree of motion is considered, under an ideal motor driver, a servo motor in the joint space can be represented as follows (16).

$$J\ddot{\theta} = T_m - T_l \quad (16)$$

Where;

T :*Motor torque*

T_l :*Load torque*

J :*Motor Inertia*

$\ddot{\theta}$:*Angular acceleration*

The load torque is considered as,

$$T_l = T_{\text{int}} + T_{ext} + (T_f + B\dot{\theta}) \quad (17)$$

T_{int} the inertial torque, derived from the Lagrange motion equations is consists of inertia torque and gravity effect. T_{ext} consists of the torque external to the system. Friction term is the sum of coulomb and viscosity terms. For a high gain current controller, input current can be assumed as the reference current. Therefore, for a DC servo motor,

$$J\ddot{\theta} = K_tI_f - \left(T_{\text{int}} + T_{ext} + T_f + B\dot{\theta}\right) \quad (18)$$

Above equation has two parameters namely J and torque constant K_t. Inertia can be changed due to the mechanical configuration of the system.

$$J = J_n + \Delta J \quad (19)$$

Similarly parameter K_t may change

$$K_t = K_{tn} + \Delta K_t \quad (20)$$

Where J_n and K_{tn} are nominal inertia and the nominal torque constant of the motor respectively.

Disturbance torque T_{dis} is represented as,

$$T_{dis} = T_l + \Delta J\ddot{\theta} - \Delta K_tI_{ref} \quad (21)$$

Dynamic equation yields,

$$(J_n + \Delta J)\ddot{\theta} = (K_{tn} + \Delta K_t)I_{ref} - T_l \quad (22)$$

By rearranging,

$$J_n\ddot{\theta} = K_{tn}I_{ref} - T_{dis} \quad (23)$$

Thus T_{dis} can be calculated as follows.

$$T_{dis} = K_{tn}I_a^{ref} - J_n\ddot{\theta} \quad (24)$$

Equation (24) can be represented by control block shown in Fig. 7.

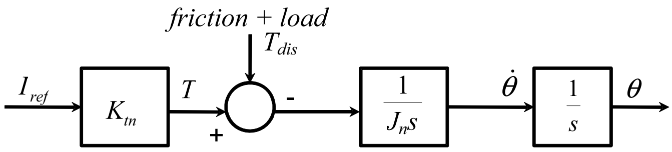

Figure 7. Motor with Disturbance Torque

In equation (24), the unknown values of the left side can be derived using the right side, which is calculable. This structure includes a differentiator which may increase the unnecessary noise in the system.

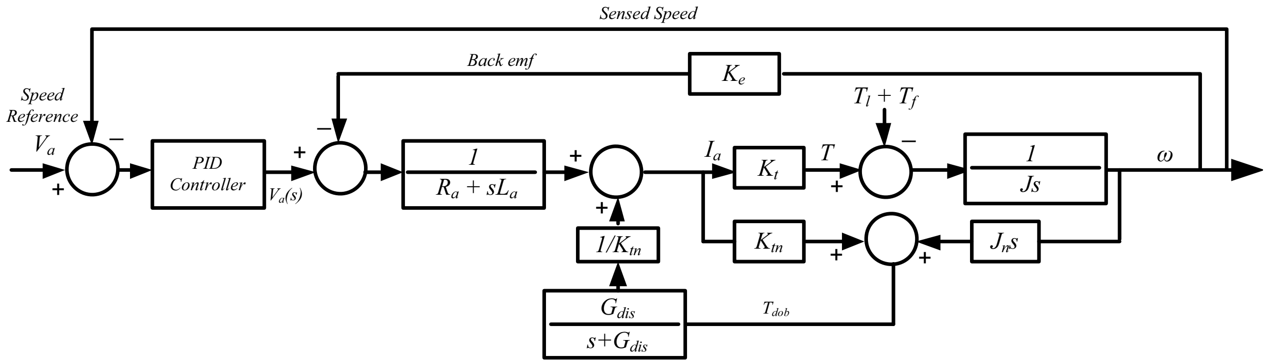

Figure 8. PID velocity conroller with DOB

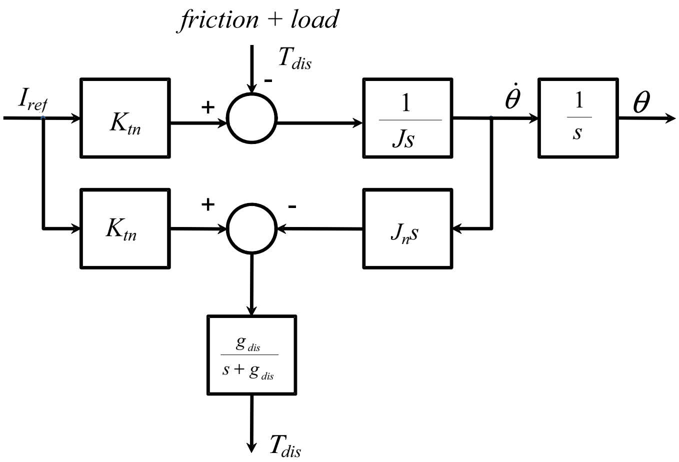

Figure 9. Disturbance based on acceleration with lowpass filter

Motor disturbance includes the frictional effects, load and the effects of parameter variations [5] as shown in equation (24). Usually this disturbance is not measurable. However if a control block as shown in Fig. 9 is implemented, the motor disturbance (T_{dis}) will become measurable and it is usually taken out after a low pass filter as shown in Fig. 9.g_{dis} is the angular cutoff frequency of the low pass filter. Disturbance observer observes the disturbance force in the system without using force sensors. It is designed such a way that it is possible to estimate the disturbance with the help of a encoder and a current sensor [6][7]. Fig. 9 shows a disturbance observer with a motor. Upper part corresponds to the motor. System represented by Fig. 9 is derived after introducing the pseudo derivative [8][9] and a low-pass filter which would improve the performance significantly. Estimated T_{dis} is feedback to compensate for unknown disturbances after converting it to a current multiplying by $1/K_{tn}$. Fig 8 shows the PID based velocity controller with disturbance observer. This is the block diagram of the system which was used for the experiment to validate the proposed system.

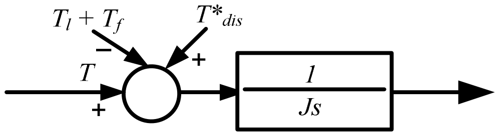

Figure 10. Disturbance compensating Torque in system equivalent with superposition

As shown in fig. 8 Disturbance torque T_{dob} derived by the disturbance observer is fed back to the system as a disturbance compensation current. However it can be seen as a disturbance compensating torque at the motor's torque output which is shown in fig. 11. However the effect of the low pass filter has been neglected in the analysis.

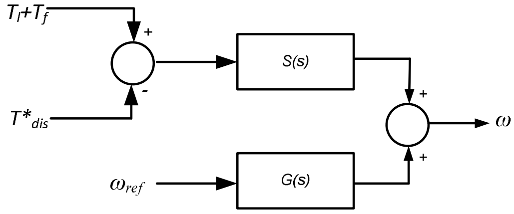

Figure 11. System equivalent with DOB

Where,

$$T^*_{dis} = \frac{(I_a K_{tn} - \omega J_n s)K_t}{K_{tn}}$$

Therefore the total disturbance acting on the system when the disturbance observer is employed is given by (25) and (26).

$$T^*_{dis} - (T_l + T_f) = \frac{(I_a K_{tn} - \omega J_n s)K_t}{K_{tn}} - (I_a K_t - \omega J s) \quad (25)$$

$$T^*_{dis} - (T_l + T_f) = \omega s\left(J - \frac{J_n K_t}{K_{tn}}\right) \quad (26)$$

If the parameters values used for K_{tn} and J_n is close to K_t and then resultant disturbance acting on the system is close zero. So the effect caused by the disturbance at the output is minimized by a great amount.

III. RESULTS

The system was tested under various types of disturbances with and without the disturbance observer. First an opposing disturbance was applied to the system.

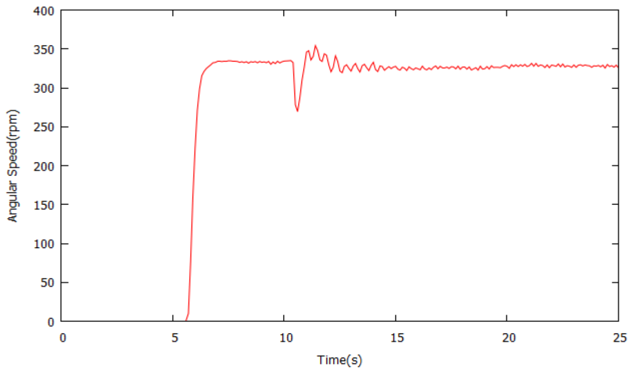

Figure 12. Opposing disturbance without DOB

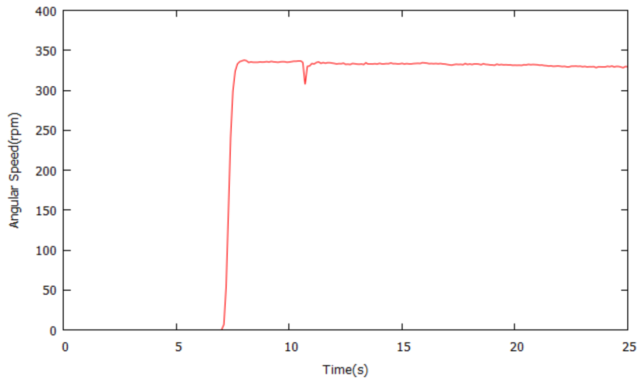

Figure 15. Opposing disturbance with DOB

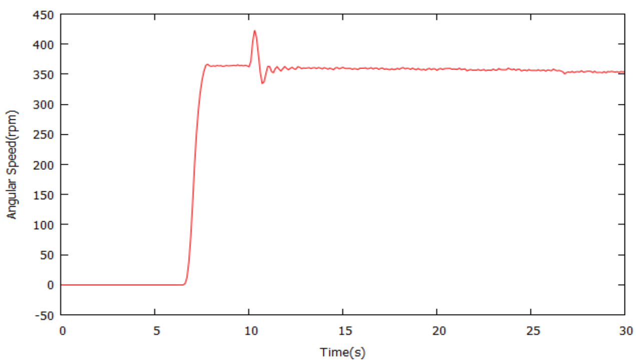

Figure 13. Supporting disturbance without DOB

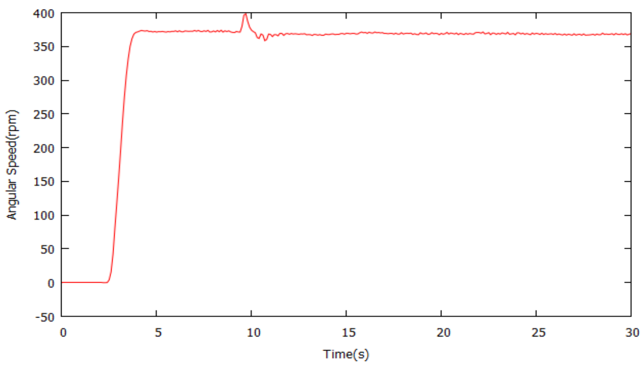

Figure 16. Supporting disturbance with DOB

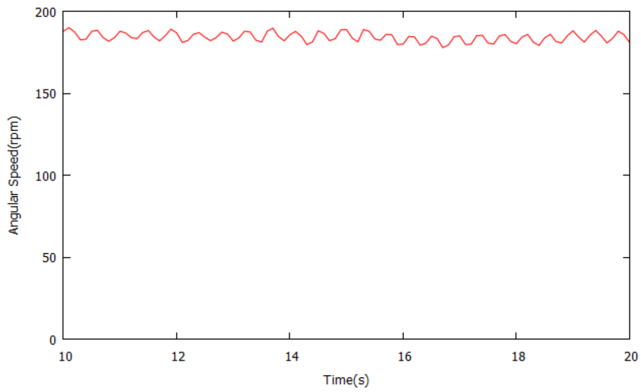

Figure 14. Periodic disturbance without DOB

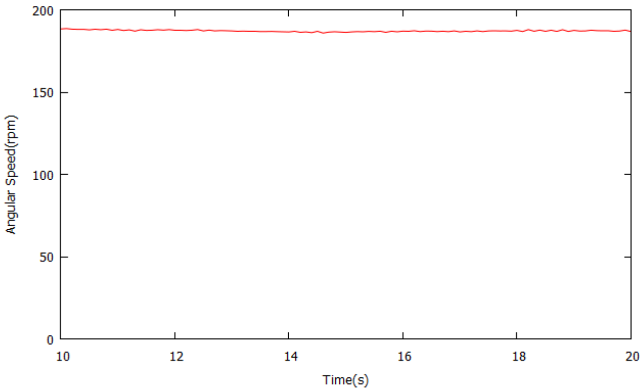

Figure 17. Periodic disturbance with DOB

Fig.12 andFig. 15 shows the behaviour of the system when an opposing disturbance is applied with and without the disturbance observer. Then a supporting disturbance was applied to the system and Fig. 13 and Fig.16 shows the behaviour of the system with and without the disturbance observer. Finally periodic disturbance was applied to the system using another DC motor connected to the same shaft. Fig. 14. And Fig. 17 shows the responses to a periodic disturbance with and without the disturbace observer.

IV . Conclusion

In this paper we proposed a velocity controlling mechanism that can be applied to brushed DC motors. Different velocity controlling arrangements were analyzed and finally the Disturbance observer based velocity controller was introduced and analyzed. A better response could be generated when velocity controllers are used with the Disturbance Observer. This method can be very useful in applications where DC motor is highly nonlinear. Since disturbance observer compensates for load variations, friction and modeling errors, system response has become highly robust as seen in the results.

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