**Disturbance Observer based Friction Compensator for a DC Motor**

M. K. C. Dinesh Chinthaka  
Department of Electrical Engineering  
University of Moratuwa  
Moratuwa, Sri Lanka  
dinesh@elect.mrt.ac.lk

R. U. G. Punchihewa  
Department of Electrical Engineering  
University of Moratuwa  
Moratuwa, Sri Lanka  
ranganafox@gmail.com

A.M.Harsha S. Abeykoon  
Department of Electrical Engineering  
University of Moratuwa  
Moratuwa, Sri Lanka  
harsha@elect.mrt.ac.lk

Abstract—Friction is primary disturbance in motion control. Different types of friction cause diminution of original torque in a DC motor, such as static friction, viscous friction etc. By some means if those can be determined and compensated, the friction effect from the DC motor can be neglected. It would be a great advantage for control systems. Authors have determined the types of frictions as well as frictional coefficients and suggested a unique way of compensating the friction in a DC motor using Disturbance Observer Method which is used to determine the disturbance torques acting on a DC motor. In simulation approach, the method is modelled using MATLAB and the results have been obtained and analysed. The block diagram consists with DC motor model with DOB and RTOB. Practical approach of the implemented block diagram is shown by the obtained results. It is discussed the possibility of applying this to real life applications.

**Keywords:** Disturbance observer, Friction compensation

1. INTRODUCTION

In the field of automation, two communal problems which are faced by researchers and engineers are variation of friction forces and variation of inertia. As far as friction term is concerned, by definition the “Friction” term stands for the resistance to relative motion between two contacting bodies [1]. Since friction dictates terms in motion control applications which will influence to the design criteria and hardware configurations especially to results, it is imperative to ponder the possibility of neutralizing friction forces somehow. Typically, the processes of non-linear friction forces are unavoidable in high performance motion control systems. One simple solution would be to use high quality mechanical components such as DC Motors. But those are costly which escalates the cost of the application way beyond the desired limit and yet the effect is not disinterested comprehensively. Another method would be to use appropriate friction compensation method with relevant computing technology and combined with good mechanical components[2]. This will remove the friction effect. The core of this research is to provide an appropriate friction compensation method for a particular DC Motor and analyzing the outcome.

Here in this research, Friction compensation is based on a reasonable accurate model for nonlinear friction which can be obtained by off-line experiment. However, it is very hard to compensate for some friction components since those tend to change when the velocity, system overall load and configuration change. So the most viable thing to do is find the friction coefficients initially using off-line experiment. Normally, those coefficients don’t change in a big tolerance. Nevertheless, there is a slight temperature and humidity effect for coefficients which can be neglected.

The aim of the research is to provide a new scheme to effectively compensate friction for a DC motor with guaranteed high precision and smoothness of motion. In this paper, Disturbance Observer (DOB) as well as Reaction Torque Observer (RTOB) prior to PID is used to compensate the frictional effects.

II. DC MOTOR MODELING

DC motor is the modest machine which converts electrical energy to the mechanical energy. Electrical equivalent model of a DC motor is shown in Fig.1

![Fig.1. Electrical model of a DC Motor](image)

A major fraction of the torque generated in the rotor of the motor is available to drive an external load. DC motors are widely used in numerous control applications, including robot manipulators, disk drive, machine tools and servo actuators and high torque application etc. DC motor is very popular especially in precise motion control applications, as the modeling is much simple than all the other motors, Fig.1 shows the electrical model of a DC motor where $R_a$ and $L_a$ represents the armature resistance and inductance respectively[3].

Applying Kirchhoff’s voltage law to the DC motor model, following differential equation (1) can be obtained.

$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + E_b$$  
(1)

Where $E_b$ is the back emf (2).

$$E_b = K \omega(t)$$  
(2)

Where $K$ is the back emf constant, then equation (1) becomes equatno. (3)

$$V_a = L_a \frac{dI_a}{dt} + R_a I_a + K \omega(t)$$  
(3)
If the total mechanical output given by the motor is $T_m$, $I_a$ can be written as equation no.(4)

$$I_a = \frac{T_m}{K_t}$$  \hspace{1cm} (4)

Where $K_t$ is the torque constant of the motor.

Applying (4) in (3) gives (5),

$$V_a = I_a \frac{d(T_m / K_t)}{dt} + R_i \frac{T_m}{K_t} + K_t \omega(t)$$  \hspace{1cm} (5)

Motor torque $T_m$ can be written as follows (6),

$$T_m = J \frac{d\omega}{dt} + B \omega + T_f + T_f$$  \hspace{1cm} (6)

Where,

- $T_f$: Static Friction
- $T_l$: Load Torque
- $T_m$: Motor Torque
- $J$: Inertia Constant
- $B$: Viscosity coefficient
- $\omega$: Rotor Speed

Equation (5) can be written in Laplace domain as (7) and Equation (4) can be written in Laplace domain as (9)

$$\omega(s) = \frac{T(s) - [T_f(s) + T_f(s)]}{Js + B}$$ \hspace{1cm} (7)

$$T(s) = K_t I_a(s)$$ \hspace{1cm} (8)

### III. MOTOR FRICTION

Friction is the tangential reaction force between two contact surfaces. In dry sliding contacts between flat surfaces, friction can be modeled as elastic and plastic deformation forces of microscopical asperities. In dry rolling contact, friction is the result of a non-symmetric pressure distribution in the contacts. When a lubricant is added, other physical variations also will appear to the total system. For low velocities, the lubricant acts as a surface film, where the shear strength determines the friction. At higher velocities and low pressures a fluid layer of lubricant is built up in the surface due to hydrodynamic effects the behavior of friction has been extensively examined during the 20th century. The experiments have been performed under idealized conditions with clean surfaces and for stationary conditions, e.g. constant velocity. Friction is a linear combination of Coulomb friction, stiction, viscous friction, and stribeck effect[4].

#### A. Static Friction

Static friction is the friction when the two surfaces are sticking. The force required to overcome the static friction and initiate motion is called the breakaway force. The static friction is usually larger than Coulomb friction force.

#### B. Coulomb Friction

Coulomb friction is independent of velocity and is always present. This friction component is dependent only on the direction of motion, in such way that it is in the direction opposite to the velocity. The magnitude of Coulomb friction depends on the properties of the surfaces in contact and the normal force. Coulomb friction is also known as kinetic friction.

#### C. Viscous Friction

Viscous friction depends on the velocity. At zero velocity the viscous friction is zero and the viscous component increases with the increase of velocity.

### IV. DETERMINING FRICTION COEFFICIENTS

To estimate the friction components, constant angular velocity motion test is performed. Here disturbance observer is used as a torque sensor, which identifies all parameters included in the motion equation by using the estimated disturbance torque of the observer. Under the constant angular velocity motion test, the disturbance observer of a DC motor estimates the friction effect.

The disturbance observer structure is given as in equation (9). In the constant angular velocity motion test, the velocity is constant and the acceleration is zero, thus there is no effect of inertia. The torque constant coefficient calculated is equal to the nominal value and since the test is conducted under the no-load condition, there is no external torque affecting to the system and $T_{int}$. Interactive torque is assumed to be negligible effect on the system. Now the observer structure will be,

$$T_{dist} = B \dot{\theta} + T_f$$ \hspace{1cm} (9)

At first to calculate the friction components of the DC motor the test was conducted with different velocities ranging from +1500 rpm to -1500 rpm which is shown is Fig.2. To obtain the better velocity response, the system used is tuned by velocity controller with disturbance observer. The disturbance torque was measured by the disturbance observer for different velocities.

![Fig.2. Friction variation with Speed in a DC Motor](image-url)
V. FRICTION COMPENSATOR MODELING

Fig. 3 shows the disturbance torque block. Here $I_{a}^{\text{ref}}$ is the torque current command determined by the DC motor motion controller. $K_r$ is the torque constant which may vary according to the rotor position. The variation of the torque constant due to the space harmonics of the flux distribution generates the motor torque pulsation. $\tau_a$ is the generated motor torque obtained by the product of $I_{a}^{\text{ref}}$ and $K_r$. The actual inertia constant $J$ changes from the nominal inertia constant $J_n$, according to the position of the rotor or after manufacturing modifications. The load torque includes $T_{\text{int}}$, the interactive torque which is defined in this research to have the elements of the pay load term, the coriolis term including the centrifugal term, and the coupled inertia term. Obviously $T_{\text{int}}$ is defined not to include the self-inertia variation torque. $T_{\text{ext}}$ is the reaction torque when mechanical system does the torque task. The friction has the components of both the static friction and the viscosity. $T_f$ is the static friction torque and $B$ is the viscosity coefficient. The disturbance torque is defined to have all the above components[5].

$$T_{\text{dis}} = T_{\text{int}} + T_{\text{ext}} + (J - J_n)\dot{\theta} + B\dot{\theta} + T_f + (K_m - K_r)I_{a}^{\text{ref}}$$ (10)

Here,
- $T_{\text{int}}$: Interactive torque
- $T_{\text{ext}}$: Reactive torque in torque task
- $f$: Static Friction
- $I_{a}^{\text{ref}}$: Torque current
- $J$: Inertia constant
- $J_n$: Nominal Inertia constant
- $B$: Viscosity coefficient
- $k_t$: Torque constant
- $g_{\text{rec}}$: RTOB filter coefficient
- $k_{\text{tn}}$: Nominal Torque constant
- $\dot{\theta}$: Angular velocity
- $s$: Laplace operator

Equation (11) and Equation (12) represent variation of inertia and motor constant respectively.

$$\Delta J = J_n - J \quad (11)$$
$$\Delta K = K_{\text{tn}} - K_t \quad (12)$$

In the robust control, it is essential to suppress the effect of the disturbance torque. For this purpose, the observer is introduced to identify it. From the above discussion the disturbance torque is defined as equation (10) to include the nonlinear effect of the load torque[6][7].

$$T_{\text{dis}} = T_{\text{int}} + T_{\text{ext}} + (J - J_n)\dot{\theta} + B\dot{\theta} + T_f + (K_m - K_r)I_{a}^{\text{ref}}$$ (10)

It is difficult to realize Fig.4 directly, because the acceleration is not detected by ordinary sensors. Instead of that, the acceleration signal is calculated through the differentiation of the velocity of the encoder. In this calculation process, a low pass filter should be inserted to suppress high frequency noise. So that’s the reason why the block diagram in Fig.5 has been used for the further modifications.

The disturbance torque is directly calculated from the acceleration of the motor and the real torque current. Here $g_{\text{rec}}$ is the angular cutoff frequency of the low-pass filter.
Disturbance observer observes the disturbance force in the system without using force sensors. It is designed such a way that it is possible to estimate the disturbance from the observer.

Fig. 6 shows the block diagram of the RTOB. Here, \( \Delta J \) is the self-inertia variation and \( \Delta K_t \) is the self-motor constant variation. To estimate only the external torque, the torque exerted on the hardware setup due to friction component should be reduced. Therefore, in order to estimate only the external torque RTOB is used\(^8\)[9].

Normally DOB and RTOB are very effective ways of finding disturbance torque values in current terms. Moreover disturbance torques can be calculated using those current values. But here both DOB and RTOB are included in given friction compensator (Fig. 7). This system makes a perfect disturbance torque estimator.

VI. RESULTS

A. Stability Analysis

This paper analyzes the stability of the friction compensator system for Fig. 8, as a basic control system of motion control. In order to do so, the simplified Transfer function (13) has been obtained in-between, the input is to be \( T_{dis} \) and output is to be \( \dot{\theta} \) while the parameters as to be in table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Motor inertia</td>
<td>( J_n )</td>
<td>0.000091</td>
<td>kgm(^2)</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>( J )</td>
<td>0.000091</td>
<td>kgm(^2)</td>
</tr>
<tr>
<td>Nominal Torque coefficient</td>
<td>( K_t )</td>
<td>0.134</td>
<td>Nm/A</td>
</tr>
<tr>
<td>Torque coefficient</td>
<td>( K_t )</td>
<td>0.134</td>
<td>Nm/A</td>
</tr>
<tr>
<td>Proportional constant</td>
<td>( K_p )</td>
<td>0.001</td>
<td>rad/s</td>
</tr>
<tr>
<td>Integral constant</td>
<td>( K_i )</td>
<td>0.0001</td>
<td>rad/s</td>
</tr>
<tr>
<td>Derivative constant</td>
<td>( K_d )</td>
<td>0.00001</td>
<td>rad/s</td>
</tr>
<tr>
<td>Cut-off frequency of low pass filter</td>
<td>( \omega_{rec} )</td>
<td>200.0</td>
<td>Hz</td>
</tr>
<tr>
<td>Viscous friction coefficient</td>
<td>( B )</td>
<td>0.0000021</td>
<td>Nm/rpm</td>
</tr>
<tr>
<td>Static friction</td>
<td>( T_f )</td>
<td>0.0043119</td>
<td>Nm</td>
</tr>
</tbody>
</table>
Suggested method gives a simplified transfer function and for stability analyzing purpose, the root locus method is used and it gives us only one root which is $-1.396 \times 10^{-21}$ root locus plot is given as in Fig.9. Apparently, it doesn’t give oscillation part but only the exponential part. Since it’s moving along the x axis towards positive infinity system is stable.

\[
\frac{-1.099}{s+1.396 \times 10^{-21}}
\]

(13)

B. Simulation Results

The stability of the system was simulated for the Fig.8 under various types of disturbances. Initially a pulsive external torque has been applied to the rotor without friction compensator. It hardly rotates one revolution. Theoretically when an external pulse is applied to the rotor, it can be considered as $T_{ex}$ and rotor will accelerate to a certain speed and continue to rotate in the same speed since the $T_{dis}$ has been removed from the motor using the friction compensator. It is show in Fig.10 and Fig.11

C. Practical Results

Practical scenario for the above simulation is carried out using the parameters shown in table I and it is shown in Fig.14. Here using a practical hardware setup actual results can be obtained. Fig.14 shows the response to single external torque pulse. As soon as the pulse is applied rotor starts to accelerate to a certain speed. If the system is adjusted correctly, smooth out put can be seen here.

In the next scenario, two way pulses have been applied to the DC motor rotor manipulator. First clockwise external torque pulse has been applied and as in the previous scenario. It starts to accelerate to a certain speed and will rotate in the constant speed afterwards. And after few seconds the direction of the pulse has been reversed. And the equivalent torque is same as initial pulse. So theoretically rotor should decelerate and speed should become zero. It is drawn in Fig. 12 and Fig. 13 respectively.
VII. CONCLUSION

In this paper authors proposed a method for friction compensation that can be applied to brushed or brushless DC motors. Theoretically, this method can be simulated and will show the viable results which can be expected. System has been theoretically simulated under various types of disturbances. Using solid practical results, the method has been evaluated possibility of using in practical applications. As far as practical scenario is concerned, there were problems aroused such as adjusting PID values. As for the results, the method is pretty much effective way of compensating the friction in a DC motor.

ACKNOWLEDGMENT

This “Verifying Disturbance Observer and PID based Friction Compensator using Practical Results” was supported by university of Moratuwa Senate Research Grant Number SRC/LT/2013/12.

REFERENCES